EXERCISES – BASS-SERRE THEORY

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Exercise 1. Verify that the different definitions we gave for free products (with or without amalgamation) are all equivalent.

Exercise 2. Factor groups inject to the free product.

Exercise 3. Describe a normal form in the HNN extension $A*_C$ (Britton's lemma).

Exercise 4. Show that the dumbbell construction $K = K_A \amalg (K_C \times I) \amalg K_B / \sim$ gives the free product with amalgamation, namely $\pi_1(K) = A *_C B$.

Exercise 5. Show that $K = K_A \amalg (K_C \times I)/ \sim$ gives the HNN extension, namely $\pi_1(K) = A*_C$.

Exercise 6. The quotient graph resulting from $\widetilde{K_{\mathcal{G}}}$ (by crushing vertex spaces and edge spaces) is a tree, and G acts on it.

Exercise 7. Prove Serre's Lemma: If $g_1, \ldots, g_n \in \mathrm{Aut}(T)$ are elliptic (have a fixed point) and all products $g_i^{\pm 1} g_j^{\pm 1}$ are elliptic, then $g_1, \ldots, g_n$ have a common fixed point.